The Classical Groups Their Invariants And Representations Princeton Landmarks In Mathematics And Physics

Geometric group theory is the study of the interplay between groups and the spaces they act on, and has its roots in the works of Henri Poincaré, Felix Klein, J.H.C. Whitehead, and Max Dehn. Office Hours with a Geometric Group Theorist brings together leading experts who provide one-on-one instruction on key topics in this exciting and relatively new field of mathematics. It's like having office hours with your most trusted math professors. An essential primer for undergraduates making the leap to graduate work, the book begins with free groups—actions of free groups on trees, algorithmic questions about free groups, the ping-pong lemma, and automorphisms of free groups. It goes on to cover several large-scale geometric invariants of groups, including quasi-isometry groups, Dehn functions, Gromov hyperbolicity, and asymptotic dimension. It also delves into important examples of groups, such as Coxeter groups, Thompson's groups, right-angled Artin groups, lamplighter groups, mapping class groups, and braid groups. The tone is conversational throughout, and the instruction is driven by examples. Accessible to students who have taken a first course in abstract algebra, Office Hours with a Geometric Group Theorist also features numerous exercises and in-depth projects designed to engage readers and provide jumping-off points for research projects.

Symmetry is a key ingredient in many mathematical, physical, and biological theories. Using representation theory and invariant theory to analyze the symmetries that arise from group actions, and with strong emphasis on the geometry and basic theory of Lie groups and Lie algebras, Symmetry, Representations, and Invariants is a significant reworking of an earlier highly-acclaimed work by the authors. The result is a comprehensive introduction to Lie theory, representation theory, invariant theory, and algebraic groups, in a new presentation that is more accessible to students and includes a broader range of applications. The philosophy of the earlier book is retained, i.e., presenting the principal theorems of representation theory for the classical matrix groups as motivation for the general theory of reductive groups. The wealth of examples and discussion prepares the reader for the complete arguments now given in the general case. Key Features of Symmetry, Representations, and Invariants: (1) Early chapters suitable for honors undergraduate or beginning graduate courses, requiring only linear algebra, basic abstract algebra, and advanced calculus; (2) Applications to geometry (curvature tensors), topology (Jones polynomial via symmetry), and combinatorics (symmetric group and Young tableaux); (3) Self-contained chapters, appendices, comprehensive bibliography; (4) More than 350 exercises (most with detailed hints for solutions) further explore main concepts; (5) Serves as an excellent main text for a one-year course in Lie group theory; (6) Benefits physicists as well as mathematicians as a reference work.

'Operads are powerful tools, and this is the book in which to read about them' - ""Bulletin of the London Mathematical Society"". Operads are mathematical devices that describe algebraic structures of many varieties and in various categories. Operads are particularly important in categories with a good notion of 'homotopy', where they play a key role in organizing hierarchies of higher homotopies. Significant examples from algebraic topology first appeared in the sixties, although the formal definition and appropriate generality were not forged until the seventies. In the nineties, a renaissance and further development of the theory were inspired by the discovery of new relationships with graph cohomology, representation theory, algebraic geometry, derived categories, Morse theory, symplectic and contact geometry, combinatorics, knot theory, moduli spaces, cyclic cohomology, and, last but not least, theoretical physics, especially string field theory and deformation quantization. The book contains a detailed and comprehensive historical introduction describing the development of operad theory from the initial period when it was a rather specialized tool in homotopy theory to the present when operads have a wide range of applications in algebra, topology, and mathematical physics. Many results and applications currently scattered in the literature are brought together here along with new results and insights. The basic definitions and constructions are carefully explained and include many details not found in any of the standard literature.

The primary goal of this 2003 book is to give a brief introduction to the main ideas of algebraic and geometric invariant theory. It assumes only a minimal background in algebraic geometry, algebra and representation theory. Topics covered include the symbolic method for computation of invariants on the space of homogeneous forms, the problem of finite-generatedness of the algebra of invariants, the theory of covariants and constructions of categorical and geometric quotients. Throughout, the emphasis is on concrete examples which originate in classical algebraic geometry. Based on lectures given at University of Michigan, Harvard University and Seoul National University, the book is written in an accessible style and contains many examples and exercises. A novel feature of the book is a discussion of possible linearizations of actions and the variation of quotients under the change of linearization. Also includes the construction of toric varieties as torus quotients of affine spaces.

The author discusses symmetric, full linear, orthogonal, and symplectic groups and determines their different invariants and representations. Using basic concepts from algebra, he examines the various properties of the groups. The book also covers topics such as matrix algebras, semigroups, commutators, and spinors, which are important in understanding the group-theoretic structure of quantum mechanics.

This is an updated English translation of Cohomologie Galoisienne, published more than thirty years ago as one of the very first versions of Lecture Notes in Mathematics. It includes a reproduction of an influential paper by R. Steinberg, together with some new material and an expanded bibliography.

Presents an updated version of Weyl's invariant theory of the classical groups, together with many of the important recent developments.

This is the first book to provide a comprehensive overview of foundational results and recent progress in the study of random matrices from the classical compact groups, drawing on the subject's deep connections to geometry, analysis, algebra, physics, and statistics. The book sets a foundation with an introduction to the groups themselves and six different constructions of Haar measure. Classical and recent results are then presented in a digested, accessible form, including the following: results on the joint distributions of the entries; an extensive treatment of eigenvalue distributions, including the Weyl integration formula, moment formulae, and limit theorems and large deviations for the spectral measures; concentration of measure with applications both within random matrix theory and in high dimensional geometry; and results on characteristic polynomials with connections to the Riemann zeta function. This book will be a useful reference for researchers and an accessible introduction for students in related fields.

This book covers the modular invariant theory of finite groups, the case when the characteristic of the field divides the order of the group, a theory that is more complicated than the study of the classical non-modular case. Largely self-contained, the book develops the theory from its origins up to modern results. It explores many examples, illustrating the theory and its contrast with the better understood non-modular setting. It details techniques for the computation of invariants for many modular representations of finite groups, especially the case of the cyclic group of prime order. It includes detailed examples of many topics as well as a quick survey of the elements of algebraic geometry and commutative algebra as they apply to invariant theory. The book is aimed at both graduate students and researchers—an introduction to many important topics in modern algebra within a concrete setting for the former, an exploration of a fascinating subfield of algebraic geometry for the latter.

This book provides an extensive and self-contained presentation of quantum and related invariants of knots and 3-manifolds. Polynomial invariants of knots, such as the Jones and Alexander polynomials, are constructed as quantum invariants, i.e. invariants derived from representations of quantum groups and from the monodromy of solutions to the Knizhnik-Zamolodchikov equation. With the introduction of the Kontsevich invariant and the theory of Vassiliev invariants, the quantum invariants become well-organized. Quantum and perturbative invariants, the LMO invariant, and finite type invariants of 3-manifolds are discussed. The ChernOCoSimons field theory and the WessOCoZuminoOCoWitten model are described as the physical background of the invariants. Contents: Knots and Polynomial Invariants; Braids and Representations of the Braid Groups; Operator Invariants of Tangles via Sliced Diagrams; Ribbon Hopf Algebras and Invariants of Links; Monodromy Representations of the Braid Groups Derived from the KnizhnikOCoZamolodchikov Equation; The Kontsevich Invariant; Vassiliev Invariants; Quantum Invariants of 3-Manifolds; Perturbative Invariants of Knots and 3-Manifolds; The LMO Invariant; Finite Type Invariants of Integral Homology 3-Spheres. Readership: Researchers, lecturers and graduate students in geometry, topology and mathematical physics."

Actions and Invariants of Algebraic Groups, Second Edition presents a self-contained introduction to geometric invariant theory starting from the basic theory of affine algebraic groups and proceeding towards more sophisticated dimensions." Building on the first edition, this book provides an introduction to the theory by equipping the reader with the tools needed to read advanced research in the field. Beginning with commutative algebra, algebraic geometry and the theory of Lie algebras, the book develops the necessary background of affine algebraic groups over an algebraically closed field, and then moves toward the algebraic and geometric aspects of modern invariant theory and quotients.

Hermann Weyl was one of the most influential mathematicians of the twentieth century. Viewing mathematics as an organic whole rather than a collection of separate subjects, Weyl made profound contributions to a wide range of areas, including analysis, geometry, number theory, Lie groups, and mathematical physics, as well as the philosophy of science and of mathematics. The topics he chose to study, the lines of thought he initiated, and his general perspective on mathematics have proved remarkably fruitful and have formed the basis for some of the best of modern mathematical research. This volume contains the proceedings of the AMS Symposium on the Mathematical Heritage of Hermann Weyl, held in May 1987 at Duke University. In addition to honoring Weyl's great accomplishments in mathematics, the symposium also sought to stimulate the younger generation of mathematicians by highlighting the cohesive nature of modern mathematics as seen from Weyl's ideas. The symposium assembled a brilliant array of speakers and covered a wide range of topics. All of the papers are expository and will appeal to a broad audience of mathematicians, theoretical physicists, and other scientists.

The book is a self-contained introduction to the results and methods in classical invariant theory.

Presents an innovative synthesis of methods used to study problems of equivalence and symmetry.

Asymptotic differential algebra seeks to understand the solutions of differential equations and their asymptotics from an algebraic point of view. The differential field of transseries plays a central role in the subject. Besides powers of the variable, these series may contain exponential and logarithmic terms. Over the last thirty years, transseries emerged variously as super-exact asymptotic expansions of return maps of analytic vector fields, in connection with Tarski's problem on the field of reals with exponentiation, and in mathematical physics. Their formal nature also makes them suitable for machine computations in computer algebra systems. This self-contained book validates the intuition that the differential field of transseries is a universal domain for asymptotic differential algebra. It does so by establishing in the realm of transseries a complete elimination theory for systems of algebraic differential equations with asymptotic side conditions. Beginning with background chapters on valuations and differential algebra, the book goes on to develop the basic theory of valued differential fields, including a notion of differential-henselianity. Next, H-fields are singled out among ordered valued differential fields to provide an algebraic setting for the common properties of Hardy fields and the differential field of transseries. The study of their extensions culminates in an analogue of the algebraic closure of a field: the Newton-Liouville closure of an H-field. This paves the way to a quantifier elimination with interesting consequences.

Providing a nearly complete selection of up-to-date methods and results on block invariants with respect to their defect groups, this book covers the classical theory pioneered by Brauer, the modern theory of fusion systems introduced by Puig, the geometry of numbers developed by Minkowski, the classification of finite simple groups, and various computer assisted methods. In a powerful combination, these tools are applied to solve many special cases of famous open conjectures in the representation theory of finite groups. Most of the material is drawn from peer-reviewed journal articles, but there are also new previously unpublished results. In order to make the text self-contained, detailed proofs are given whenever possible. Several tables add to the text's usefulness as a reference. The book is aimed at experts in group theory or representation theory who may wish to make use of the presented ideas in their research.

This work explores the fundamental concepts in arithmetic. It begins with the definitions and properties of algebraic fields. The theory of divisibility is then discussed. There follows an introduction to p-adic numbers and then culminates with an extensive examination of algebraic number fields.

This book is both an easy-to-read textbook for invariant theory and a challenging research monograph that introduces a new approach to the algorithmic side of invariant theory. Students will find the book an easy introduction to this "classical and new" area of mathematics. Researchers in mathematics, symbolic computation, and computer science will get access to research ideas, hints for applications, outlines and details of algorithms, examples and problems.

DIVThis in-depth introduction to classical topics in higher algebra provides rigorous, detailed proofs for its explorations of some of mathematics' most significant concepts, including matrices, invariants, and groups. 1926 edition. /div

This is the first book to deal with invariant theory and the representations of finite groups.

Geometric Invariant Theory (GIT) is developed in this text within the context of algebraic geometry over the real and complex numbers. This sophisticated topic is elegantly presented with enough background theory included to make the text accessible to advanced graduate students in mathematics and physics with diverse backgrounds in algebraic and differential geometry. Throughout the book, examples are emphasized. Exercises add to the reader’s understanding of the material; most are enhanced with hints. The exposition is divided into two parts. The first part, ‘Background Theory’, is organized as a reference for the rest of the book. It contains two chapters developing material in complex and real algebraic geometry and algebraic groups that are difficult to find in the literature. Chapter 1 emphasizes the relationship between the Zariski topology and the canonical Hausdorff topology of an algebraic variety over the complex numbers. Chapter 2 develops the interaction between Lie groups and algebraic groups. Part 2, ‘Geometric Invariant Theory’ consists of three chapters (3–5). Chapter 3 centers on the Hilbert–Mumford theorem and contains a complete development of the Kempf–Ness theorem and Vindberg’s theory. Chapter 4 studies the orbit structure of a reductive algebraic group on a projective variety emphasizing Kostant’s theory. The final chapter studies the extension of classical invariant theory to products of classical groups emphasizing recent applications of the theory to physics.

This book is based on the notes of the authors' seminar on algebraic and Lie groups held at the Department of Mechanics and Mathematics of Moscow University in 1967/68. Our guiding idea was to present in the most economic way the theory of semisimple Lie groups on the basis of the theory of algebraic groups. Our main sources were A. Borel's paper [34], C. Chevalley's seminar [14], seminar "Sophus Lie" [15] and monographs by C. Chevalley [4], N. Jacobson [9] and J-P. Serre [16, 17]. In preparing this book we have completely rearranged these notes and added two new chapters: "Lie groups" and "Real semisimple Lie groups". Several traditional topics of Lie algebra theory, however, are left entirely disregarded, e.g. universal enveloping algebras, characters of linear representations and (co)homology of Lie algebras. A distinctive feature of this book is that almost all the material is presented as a sequence of problems, as it had been in the first draft of the seminar's notes. We believe that solving these problems may help the reader to feel the seminar's atmosphere and master the theory. Nevertheless, all the non-trivial ideas, and sometimes solutions, are contained in hints given at the end of each section. The proofs of certain theorems, which we consider more difficult, are given directly in the main text. The book also contains exercises, the majority of which are an essential complement to the main contents.

The first edition of this book presented the theory of linear algebraic groups over an algebraically closed field. The second edition, thoroughly revised and expanded, extends the theory over arbitrary fields, which are not necessarily algebraically closed. It thus represents a higher aim. As in the first edition, the book includes a self-contained treatment of the prerequisites from algebraic geometry and commutative algebra, as well as basic results on reductive groups. As a result, the first part of the book can well serve as a text for an introductory graduate course on linear algebraic groups.

Lie groups has been an increasing area of focus and rich research since the middle of the 20th century. In Lie Groups: An Approach through Invariants and Representations, the author's masterful approach gives the reader a comprehensive treatment of the classical Lie groups along with an extensive introduction to a wide range of topics associated with Lie groups: symmetric functions, theory of algebraic forms, Lie algebras, tensor algebra and symmetry, semisimple Lie algebras, algebraic groups, group representations, invariants, Hilbert theory, and binary forms with fields ranging from pure algebra to functional analysis. By covering sufficient background material, the book is made accessible to a reader with a relatively modest mathematical background. Historical information, examples, exercises are all woven into the text. This unique exposition is suitable for a broad audience, including advanced undergraduates, graduates, mathematicians in a variety of areas from pure algebra to functional analysis and mathematical physics.

Sample Text

If classical Lie groups preserve bilinear vector norms, what Lie groups preserve trilinear, quadrilinear, and higher order invariants? Answering this question from a fresh and original perspective, Predrag Cvitanovic takes the reader on the amazing, four-thousand-diagram journey through the theory of Lie groups. This book is the first to systematically develop, explain, and apply diagrammatic projection operators to construct all semi-simple Lie algebras, both classical and exceptional. The invariant tensors are presented in a somewhat unconventional, but in recent years widely used, "birdtracks" notation inspired by the Feynman diagrams of quantum field theory. Notably, invariant tensor diagrams replace algebraic reasoning in carrying out all group-theoretic computations. The diagrammatic approach is particularly effective in evaluating complicated coefficients and group weights, and revealing symmetries hidden by conventional algebraic or index notations. The book covers most topics needed in applications from this new perspective: permutations, Young projection operators, spinorial representations, Casimir operators, and Dynkin indices. Beyond this well-traveled territory, more exotic vistas open up, such as "negative dimensional" relations between various groups and their representations. The most intriguing result of classifying primitive invariants is the emergence of all exceptional Lie groups in a single family, and the attendant pattern of exceptional and classical Lie groups, the so-called Magic Triangle. Written in a lively and personable style, the book is aimed at researchers and graduate students in theoretical physics and mathematics.

This book gives a unified, complete, and self-contained exposition of the main algebraic theorems of invariant theory for matrices in a characteristic free approach. More precisely, it contains the description of polynomial functions in several variables on the set of matrices with coefficients in an infinite field or even the ring of integers, invariant under simultaneous conjugation. Following Hermann Weyl's classical approach, the ring of invariants is described by formulating and proving (1) the first fundamental theorem that describes a set of generators in the ring of invariants, and (2) the second fundamental theorem that describes relations between these generators. The authors study both the case of matrices over a field of characteristic 0 and the case of matrices over a field of positive characteristic. While the case of characteristic 0 can be treated following a classical approach, the case of positive characteristic (developed by Donkin and Zubkov) is much harder. A presentation of this case requires the development of a collection of tools. These tools and their application to the study of invariants are exlained in an elementary, self-contained way in the book.

Multiplicative invariant theory, as a research area in its own right within the wider spectrum of invariant theory, is of relatively recent vintage. The present text offers a coherent account of the basic results achieved thus far.. Multiplicative invariant theory is intimately tied to integral representations of finite groups. Therefore, the field has a predominantly discrete, algebraic flavor. Geometry, specifically the theory of algebraic groups, enters through Weyl groups and their root lattices as well as via character lattices of algebraic tori. Throughout the text, numerous explicit examples of multiplicative invariant algebras and fields are presented, including the complete list of all multiplicative invariant algebras for lattices of rank 2. The book is intended for graduate and postgraduate students as well as researchers in integral representation theory, commutative algebra and, mostly, invariant theory.

In this renowned volume, Hermann Weyl discusses the symmetric, full linear, orthogonal, and symplectic groups and determines their different invariants and representations. Using basic concepts from algebra, he examines the various properties of the groups. Analysis and topology are used wherever appropriate. The book also covers topics such as matrix algebras, semigroups, commutators, and spinors, which are of great importance in understanding the group-theoretic structure of quantum mechanics. Hermann Weyl was among the greatest mathematicians of the twentieth century. He made fundamental contributions to most branches of mathematics, but he is best remembered as one of the major developers of group theory, a powerful formal method for analyzing abstract and physical systems in which symmetry is present. In The Classical Groups, his most important book, Weyl provided a detailed introduction to the development of group theory, and he did it in a way that motivated and entertained his readers. Departing from most theoretical mathematics books of the time, he introduced historical events and people as well as theorems and proofs. One learned not only about the theory of invariants but also when and where they were originated, and by whom. He once said of his writing, "My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful." Weyl believed in the overall unity of mathematics and that it should be integrated into other fields. He had serious interest in modern physics, especially quantum mechanics, a field to which The Classical Groups has proved important, as it has to quantum chemistry and other fields. Among the five books Weyl published with Princeton, Algebraic Theory of Numbers inaugurated the Annals of Mathematics Studies book series, a crucial and enduring foundation of Princeton's mathematics list and the most distinguished book series in mathematics.

The articles in this volume cover a broad range of topics in algebraic geometry: classical varieties, linear system, birational geometry, Minimal Model Program, moduli spaces, toric varieties, enumerative theory of singularities, equivariant cohomology and arithmetic questions.

"This monograph weaves together fundamentals of Mikhail Leonidovich Gromov's hyperbolic groups with the theory of cube complexes dual to spaces with walls. Many fundamental new ideas and methodologies are presented here for the first time: A cubical small-cancellation theory generalizing ideas from the 1960's, a version of "Dehn Filling" that works in the category of special cube complexes, and a variety of new results about right-angled Artin groups. The book culminates by providing an unexpected new theorem about the nature of hyperbolic groups that are constructible as amalgams. Among the stunning applications, are the virtual fibering of cusped hyperbolic 3-manifolds and the resolution of R.J. Baumslag's conjecture on the residual finiteness of one-relator groups with torsion. Most importantly, the book outlines the author's program towards the resolution of the most important remaining conjectures of William Thurston, and achieves substantial progress in this direction. This monograph, which is richly illustrated with over 100 drawings, will be of interest to graduate students and scholars working in geometry, algebra, and topology. This groundbreaking monograph, intended for the Annals of Math series, lays the mathematical groundwork for the solution of the Thurston-Haken Conjecture, a significant result in geometric group theory. It outlines one of the deepest and most surprising pieces of this result, which also has a variety of other implications for geometric group theory. This work also has applications to low-dimensional topology, and the results in this book have since been used by other mathematicians to provide other important results"--

Central to the work is the classification of the conjugation and reversion anti-involutions that arise naturally in the theory. It is of interest that all the classical groups play essential roles in this classification. Other features include detailed sections on conformal groups, the eight-dimensional non-associative Cayley algebra, its automorphism group, the exceptional Lie group G[subscript 2], and the triality automorphism of Spin 8.